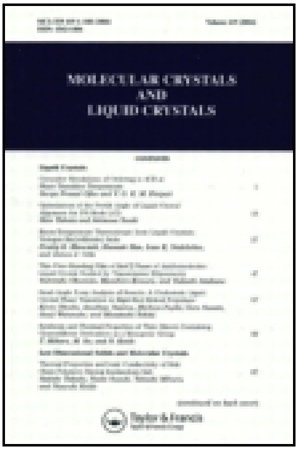

Molecular Crystals and Liquid Crystals Science and Technology. Section A. Molecular Crystals and Liquid Crystals
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gmcl19

Stress Balances In Nematic Interfaces

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Version of record first published: 27 Oct 2006

To cite this article: Alejandro D. Rey (2000): Stress Balances In Nematic Interfaces, Molecular Crystals and Liquid Crystals Science and Technology. Section A. Molecular Crystals and Liquid Crystals, 338:1, 197-206

To link to this article: http://dx.doi.org/10.1080/10587250008024430

Mol. Cryst. Liq. Cryst., Vol. 338, pp. 197-206
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Stress Balances In Nematic Interfaces

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(Received December 30, 1998; In final form May 20, 1999)

This paper presents the equations of capillary hydrostatics for interfaces between viscous isotropic fluids and nematic liquid crystals. The interfacial stress balance equation involves the surface divergence of the surface stress tensor. It is shown that the anisotropic elastic contribution due to weak anchoring give rise to bending stresses not observed in isotropic fluids. It is shown that surface gradients of the bending stresses are a source of interfacial normal forces even in planar interfaces. Surface gradients in the normal surface stresses due to weak anchoring generate interfacial tangential forces, equivalent to those present in the thermocapillarity, electrocapillarity, and diffusocapillarity of isotropic fluids. Mathematical expressions corresponding to biaxial nematics for anisotropic surface elasticity, surface stress tensor, Marangoni and normal forces are derived, characterized, and used to establish the main interfacial phenomena driven by nematic biaxial ordering.

Keywords: nematic liquid crystals; nematocapillarity; interfacial stress; normal stress equation; tangential stress equation

1. INTRODUCTION

Capillary hydrostatics in isotropic fluids is concerned with interfacial phenomena that arise between two fluids at rest such as in droplets, bubbles, fibers, and films[1,2,3]. A matter of interest is to determine the equilibrium shape of the interface, such as in liquid bridges, films on vertical solid surfaces, and dispersed droplets. In other cases the geometry of the interface is used to determine the surface tension, such as in the Wilhelmy plate [3]. In the liquid crystals field interfacial phenomena are important in applications such as in the manufacturing of polymer-dipersed liquid crystals, nematic polymer fibers and films, and nematic-thermoplastic polymer blends. In addition surface tension in liquid crystals is measured used the same techniques as in isotropic fluids[4,5]. Thus a bet-

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ter understanding of interfacial phenomena in liquid crystalline interfaces is of practical importance for product manufacturing, and property evaluation.

At the macroscopic level the theoretical framework to describe capillary hydrostatics of isothermal isotropic fluids consists of the force balance equation in the bulk phases together with the interfacial stress balance condition [1,3]. The interfacial stress boundary condition play a central role in capillary hydrostatics since it involves the interfacial shape. In this boundary condition the terms that balance the stresses in the bulk phases is the surface divergence of the surface stress tensor. As a consequence the surface stress tensor itself must describe the necessary deformation modes. There are many constitutive equations for the surface stress tensor such as elastic, viscoelastic, viscoplastic, etc [3]. Almost always the surface stress tensor is a 2×2 symmetric tensor with normal elastic stress components, and shear stress components in the presence of flow[3]. On the other hand bending stresses are not present in isotropic fluids [3]. The presence of nematic ordering introduces anisotropic elastic behaviour in the bulk nematic phase as well as anisotropic elastic modes in the interface of a nematic liquid crystal and an isotropic fluid. It will be shown below that the anisotropic surface elasticity of nematic liquid crystals introduces bending stresses not found in interfaces between isotropic fluids. Thus the conventional interface stress balance equation that are is the foundation of capillary hydrostatics [1,3] must be augmented with the nematic anisotropic elastic modes. Since bending stresses are generated even in planar interfaces, curvature free normal forces are possible in nematic-isotropic fluid interfaces. In addition since surface normal stresses are anisotropic, tangential forces may be generated even under isothermal conditions. The novel effects explored in this paper are not displayed by isotropic fluid interfaces.

The objectives of this paper are: (1) to present the interfacial stress balance equations of isothermal, incompressible, capillary nematostatics, (2) derive a constitutive equation for the surface stress tensor of nematic-isotropic fluid interfaces, and (3) derive the normal stress and tangential stress balance equation for nematic-isotropic fluid interfaces. The scope of the paper is limited to the governing equations while applications will be pursued in future work. Vector level formulations of liquid crystal interfacial phenomena have been presented [6,7], but here we generalize those results by using a tensor level formulation. The use of the tensor order parameter for interfacial problems in liquid crystals is frequent and necessary [8].

The organization of this paper is as follows. Section 2 presents the interface stress balance equations, including expressions for the bulk stresses. Section 3 derives the surface stress tensor for a nematic-isotropic fluid interface, and identifies the origin of the normal and bending stresses. Section 4 gives the normal and tangential interfacial stress balance equations, and identifies the origin of the normal and tangential surface forces. Section 5 presents a detailed analysis of the role of biaxiality on interfacial phenomena, including surface elasticity, surface stress tensor, Marangoni and normal surface forces. Section 6 gives the conclusions.

2. INTERFACIAL STRESS BALANCE EQUATION

In this section we present the interfacial stress balance equation for an interphase between an isotropic viscous fluid and a uniaxial rod-like nematic liquid crystal [9]. The system is isothermal, and both phases are incompressible. The interphase is assumed to be elastic. Assume that a nematic liquid crystal (NLC) occupies region R^-, and that an isotropic viscous fluid region R^+. The NLC structure is given by the symmetric, traceless, 3×3 tensor order parameter **Q**, usually parametrized in terms of its eigenvectors as follows: $\mathbf{Q} = S(\mathbf{nn}\text{-}\boldsymbol{\delta}/3)+P(\mathbf{mm}\text{-}\mathbf{ll})/3$, where S (P) is the uniaxial (biaxial) scalar order parameter, and **(n,m,l)** are the orthonormal eigenvectors; **n** is the director. The orientation of the interface between the (±) regions is characterized by a unit normal **k**, directed from R- into R+. The interfacial stress boundary condition at the NLC-isotropic viscous fluid interface is expressed by [3]:

$$-\mathbf{k}.\left(\mathbf{t}^{+}-\mathbf{t}^{-}\right)=\nabla_{s}.\mathbf{t}^{se};\ \mathbf{t}^{se}=\mathrm{F_s}\mathbf{I}_s+\mathbf{t}^{sd} \qquad (1a,b)$$

where $\mathbf{t}^{\pm}$ is the total stress tensor in the two (±) bulk phases, $\nabla_s = \mathbf{I}_s.\nabla$ is the surface gradient operator, $\mathbf{I}_s = \mathbf{I} - \mathbf{kk}$ is the surface idem factor, F_s is the interfacial free energy density, $\mathbf{t}^{se}$ is the surface elastic stress tensor, and $\mathbf{t}^{sd}$ is the surface distortion stress tensor.

A widely used expression for F_s is [8,10,11]:

$$\mathrm{F_s}(\mathbf{Q},\mathbf{k},\mathrm{N})=\mathrm{F_s}(0)+\beta_{11}\mathbf{k}.\mathrm{N}+\beta_{20}\mathbf{Q}:\mathbf{Q}+\beta_{21}\mathrm{N}.\mathrm{N}+\beta_{22}(\mathbf{k}.\mathrm{N})^2;\ \mathrm{N}=\mathbf{Q}.\mathbf{n} \qquad (2)$$

where F_s (0) is the usual isotropic part, $\{\beta_{ij}\}$; ij = 11,20,21,22 are phenomenological parameters (energy/area) that can be taken to be temperature independent, and where the **Q** dependent terms arise whenever the surface tensor order parameter deviates from the easy surface tensor order parameter $\mathbf{Q}^o$; $\mathbf{Q}^o$ minimizes F_s. The constant β_{11} represents the effects due to the van de Waals interactions between the nematic and the isotropic phases. The easy axis for the director may be planar, homeotropic, or tilted, according to the signs and magnitudes of the $\{\beta_{ij}\}$; ij = 11,20,21,22 [11].

In the isotropic fluid phase the stress tensor is:

$$\mathbf{t}^{+} = -\mathrm{p}^{+}\mathbf{I} \tag{3}$$

where p^{+} is the pressure, and in the nematic phase by:

$$\mathrm{t}_{\mathrm{ji}}^{-} = -\mathrm{p}^{-}\mathrm{I}_{\mathrm{ji}} + \mathrm{t}_{\mathrm{ji}}^{\mathrm{E}};\ \mathrm{t}_{\mathrm{ji}}^{\mathrm{E}} = -\frac{\partial \mathrm{F_G}}{\partial \mathrm{Q}_{\mathrm{kl,j}}}\mathrm{Q}_{\mathrm{kl,i}} \tag{4a, b}$$

where $\mathrm{F_G}$ is the Frank gradient elasticity:

$$\mathrm{F_G} = \frac{\mathrm{L}_1}{2}\mathrm{Q}_{\mathrm{ij,k}}\mathrm{Q}_{\mathrm{ij,k}} + \frac{\mathrm{L}_2}{2}\mathrm{Q}_{\mathrm{ij,j}}\mathrm{Q}_{\mathrm{ij,j}} \tag{5}$$

and $\mathbf{t}^{\mathrm{E}}$ the Ericksen stress,

$$\mathrm{t}_{\mathrm{ji}}^{\mathrm{E}} = -\mathrm{L}_1\mathrm{Q}_{\mathrm{kl,j}}\mathrm{Q}_{\mathrm{kl,i}} - \mathrm{L}_2\mathrm{Q}_{\mathrm{kj,i}}\mathrm{Q}_{\mathrm{kp,p}} \tag{6}$$

which shows that normal Ericksen stresses (i=j) and shear Ericksen stresses (i=j) may appear even in one dimensional problems such as in a twisted nematic film [12]. If isotropic viscous fluids at rest do not generate shear stresses then shear Ericksen stresses must be balanced by tangential forces generated by surface gradients of surface stress tensor. In most fluids such tangential forces may exist due to surface tension gradients driven by thermal, electric charge, and mass diffusion gradients. If we assume none of this effects to be present, we show below that surface tensor order parametr gradients provide a mechanism to generate tangential forces.

3. THE SURFACE ELASTIC STRESS TENSOR FOR NEMATIC-ISOTROPIC FLUID INTERFACES

To obtain an expression for $\mathbf{t}^{\mathrm{sd}}$ needed to use eqn.(1) we use the principle of virtual work and compute the change in free energy $\delta\, \mathrm{F_s}$ due to a small change in the unit normal $\delta\, \mathbf{k} = -\mathbf{k}.(\nabla_{\mathrm{s}}\mathbf{u})^{\mathrm{T}}$ caused by a displacement $\mathbf{u}$,

$$\delta\, \mathrm{F_s} = \int \mathbf{t}^{\mathrm{sd}} : (\nabla_{\mathrm{s}}\mathbf{u})^{\mathrm{T}} dS = -\int \mathbf{I_s}.\left[\frac{\partial \mathrm{F_s}}{\partial \mathbf{N}}.\frac{\partial \mathbf{N}}{\partial \mathbf{k}} + \frac{\partial \mathrm{F_s}}{\partial \mathbf{k}}\right]\mathbf{k} : (\nabla_{\mathrm{s}}\mathbf{u})^{\mathrm{T}}\mathrm{dS} \tag{7}$$

where we used the facts that $\mathrm{F_s}=\mathrm{F_s}(\mathbf{k}, \mathbf{N}, \mathbf{Q})$, $\mathbf{N}(\mathbf{k},\mathbf{Q}) = \mathbf{Q}.\mathbf{k}$, and $\mathbf{I_s}.\mathbf{t}^{\mathrm{sd}} = \mathbf{t}^{\mathrm{sd}}$. The surface stress deformation tensor is given by:

$$\mathbf{t}^{sd} = -\mathbf{I_s}.\left[\frac{\partial \mathrm{F_s}}{\partial \mathbf{N}}.\frac{\partial \mathbf{N}}{\partial \mathbf{k}} + \frac{\partial \mathrm{F_s}}{\partial \mathbf{k}}\right]\mathbf{k} = -\mathbf{I_s}.\frac{\partial \mathrm{F_s}}{\partial \mathbf{N}}.\ \mathbf{Qk} - \mathbf{I_s}.\frac{\partial \mathrm{F_s}}{\partial \mathbf{k}}\mathbf{k} \tag{8}$$

Thus the surface elastic stress tensor $\mathbf{t}^{se}$ is:

$$\mathbf{t}^{se} = \mathrm{F}_s\mathbf{I}_s + \mathbf{t}^{sd} = \mathrm{F}_s\mathbf{I}_s - \mathbf{I_s}.\frac{\partial \mathrm{F_s}}{\partial \mathbf{N}}.\ \mathbf{Qk} - \mathbf{I_s}.\frac{\partial \mathrm{F_s}}{\partial \mathbf{k}}\mathbf{k} \tag{9}$$

In component form the surface elastic stress tensor $\mathbf{t}^{se}$ is given by the sum of the normal stresses $(\mathbf{t}_{11}^{se}, \mathbf{t}_{22}^{se})$ and bending stresses $(\mathbf{t}_{13}^{se}, \mathbf{t}_{23}^{se})$:

$$\mathbf{t}^{se} = \mathbf{i}_1\ \mathbf{i}_1 \mathbf{t}_{11}^{se} + \mathbf{i}_2\ \mathbf{i}_2 \mathbf{t}_{22}^{se} + \mathbf{i}_1\ \mathbf{k}\ \mathbf{t}_{13}^{se} + \mathbf{i}_2\ \mathbf{k}\ \mathbf{t}_{23}^{se} \tag{10}$$

where $(\mathbf{i}_1, \mathbf{i}_2)$ are the surface orthonormal basis vectors. The surface elastic stress tensor $\mathbf{t}^{se}$ is a 2×3 tensor. The magnitudes of the normal stresses are given by:

$$\mathbf{t}_{11}^{se} = \mathbf{t}_{22}^{se} = \mathrm{F_s}(\mathbf{k}, \mathbf{N}, \mathbf{Q}) \tag{11}$$

and of the bending streesses by:

$$\mathbf{t}_{13}^{se} = -\left[\mathbf{i}_1 . \frac{\partial \mathrm{F_s}}{\partial \mathbf{N}} . \mathbf{Q} + \mathbf{i}_1 . \frac{\partial \mathrm{F_s}}{\partial \mathbf{k}}\right]; \ \mathbf{t}_{23}^{se} = -\left[\mathbf{i}_2 . \frac{\partial \mathrm{F_s}}{\partial \mathbf{N}} . \mathbf{Q} + \mathbf{i}_2 . \frac{\partial \mathrm{F_s}}{\partial \mathbf{k}}\right] \tag{12a, b}$$

Bending stresses arise in tilted surface orientations. For example in a uniaxial state and some values of $\{\beta ij\}$ (see eqn. (2)), the magnitude of the bending stresses are a maximum when the surface director is oriented at $\pi/4$ from the unit normal and zero for planar and homeotropic director surface orientations. For isotropic viscous fluids the bending stresses are zero [3, 13].

It can be seen that surface gradients of normal stresses $(\mathbf{t}_{11}^{se}, \mathbf{t}_{22}^{se})$ generate tangential forces $\mathbf{f}_{//}$ since:

$$\mathbf{f}_{//} = \nabla_s \mathrm{F_s} . \mathbf{I}_s \tag{13}$$

The surface gradients of the interfacial tension may arise due to changes of the tensor order along the interface. Surface gradients in the bending stresses generate normal forces in addition to those arising from the usual curvature effects, $\mathrm{F_s} \nabla_s . \mathrm{I_s}$. These additional normal forces $\mathbf{f}_\perp$ arise from surface gradients of the distortion stress tensor:

$$\mathbf{f}_\perp = \nabla_s . \mathbf{t}^{sd} \tag{14}$$

4. NORMAL AND TANGENTIAL STRESS BALANCES

Replacing the expression of the surface elastic stress tensor $\mathbf{t}^{se}$ into equation (1) it is found:

$$\mathbf{f} = \nabla_s . \mathbf{t}^{se} = \left\{ \left[\frac{\mathrm{d}\gamma_{\mathrm{an}}}{\mathrm{d}\mathbf{Q}}\right]^{[s]} : (\nabla_s \mathbf{Q})^{\mathrm{T}} \right\} . \mathbf{I}_s + \left\{ 2\mathrm{H}\gamma - 2\mathrm{H}\left(\frac{\mathrm{d}\gamma_{\mathrm{an}}}{\mathrm{d}\mathbf{k}} . \mathbf{k}\right) - \nabla_s . \left(\frac{\mathrm{d}\gamma_{\mathrm{an}}}{\mathrm{d}\mathbf{k}}\right) \right\} \mathbf{k} \tag{15a}$$

where the total derivative $\mathrm{d}\gamma_{\mathrm{an}}/\mathrm{d}\mathbf{Q}$ is given by:

$$\frac{\mathrm{d}\gamma_{\mathrm{an}}}{\mathrm{d}\mathbf{Q}} = \frac{\partial \gamma_{\mathrm{an}}}{\partial \mathbf{N}} \mathbf{k} + \frac{\partial \gamma_{\mathrm{an}}}{\partial \mathbf{Q}} \tag{15b}$$

and where [s] denotes symmetric and traceless, and H is the mean surface curvature: $2\,H = -\nabla_s.\mathbf{k}$. For isotropic fluids the normal force balance equation involves the role of surface tension due to surface curvature effects, but for nematics additional effects due to anchoring effects are present. The normal force balance is obtained by projecting the vector equation (1) along the unit normal **k**:

$$-\mathbf{k}.\left(\mathbf{t}^{+}-\mathbf{t}^{-}\right).\mathbf{kk} = \nabla_s.\mathbf{t}^{se}.\mathbf{kk} = \mathbf{f}_{\perp}$$

$$\mathbf{f}_{\perp} = \left[2\mathrm{HF_s} - \left(\nabla_s \frac{\partial \mathrm{F_s}}{\partial \mathbf{N}}\right) : \mathbf{Q} - \frac{\partial \mathrm{F_s}}{\partial \mathbf{N}}.(\nabla_s.\,\mathbf{Q})^{\mathrm{T}} - \nabla_s.\left(\frac{\partial \mathrm{F_s}}{\partial \mathbf{k}}\right) - 2\mathrm{H}\left(\frac{\partial \mathrm{F_s}}{\partial \mathbf{N}}.\mathbf{N} + \frac{\partial \mathrm{F_s}}{\partial \mathbf{k}}.\mathbf{k}\right)\right]\mathbf{k} \qquad (16a,b)$$

If F_s is independent of **(k,N,Q)** the equation reduces to the static normal force Laplace balance equation for isotropic systems. The equation shows that even for a planar interface (H=0) the normal stresses in the bulk (±) phases may be unequal due to surface gradients of **Q**.

The tangential force balance equations involves gradients in the surface free energy density and is obtained by projecting equation (1) along the tangent direction:

$$-\mathbf{k}.(\mathbf{t}^{+}-\mathbf{t}^{-}).\mathbf{I}_s = \nabla_s.\mathbf{t}^{se}.\mathbf{I}_s = \mathbf{f}_{//}, \quad \mathbf{f}_{//} = \left[\left[\frac{\mathrm{dF_s}}{\mathrm{d}\mathbf{Q}}\right]^{[s]} : (\nabla_s \mathbf{Q})^{\mathrm{T}}\right].\mathbf{I}_s \qquad (17a,b)$$

The tangential Ericksen stress in the bulk nematic phase must balanced by the surface tangential force arising from surface gradients in **Q**. If there are no Ericksen shear stresses then static conditions can not exist and a Marangoni flow [1,2,3] is set in motion. The tangential force $\mathbf{f}_{//}$ is the nematic Marangoni force.

5. STRESS BALANCES FOR BIAXIAL NEMATIC INTERFACES

The previous sections 2,3,4 provided a detailed analysis of stress balances involving nematic liquid crystals, whose surface state was given in terms of the symmetric, traceless tensor order parameter Q. To provide a deeper analysis of interfacial stresses it is useful and instructive to separate the effects of the eigenvalues and eigenvectors of Q. In this last section we demonstrate the special properties that arise from biaxiality.

A common and frequent way to express the symmetric traceless tensor order parameter **Q** is:

$$\mathbf{Q} = \mu_{\mathrm{n}}\ \mathbf{nn} + \mu_{\mathrm{m}}\ \mathbf{mm} + \mu_{\mathrm{l}}\ \mathbf{ll} \qquad (18)$$

where the eigenvectors **n,m,l** are the orthonormal director triad. The eigenvalues are given by:

$$\mu_n = 2S/3;\ \mu_m = -S/3 + P/3;\ \mu_l = -S/3 - P/3 \tag{19}$$

where S is the uniaxial scalar order parameter, and P is the biaxial scalar order parameter. For biaxial nematics P. Since **Q** is traceless, $\mu_n + \mu_m + \mu_l = 0$. The eigenvalues are restricted to: $-1/3 \le \mu_i \le 2/3$; i=n,m,l. In the general case Q has five independent components. In order to exhibit the special properties of biaxial nematic interfaces without loss of physics we set the biaxial director **l** along one of the unit surface vectors, $\mathbf{l}=\mathbf{i}_2$. In this case the directors **n** and **m** are on the $(\mathbf{i}_1,\mathbf{k})$ plane, and the number of independent components of **Q** reduces to three. Introducing the tilt angle ψ between the director **n** and the unit normal **k**, the directors **n** and **m**, in the $(\mathbf{i}_1,\mathbf{i}_2,\mathbf{k})$ frame, are parametrized as:

$$\mathbf{n}(x_1,x_2) = (\sin\psi, 0, \cos\psi);\ \mathbf{m}(x_1,x_2) = (\cos\psi, 0, -\sin\psi) \tag{20a, b}$$

where (x_1,x_2) are the coordinates along $(\mathbf{i}_1,\mathbf{i}_2)$. We now proceed to express the surface free energy density F_s, the surface stress tensor $\mathbf{t}^{se}$, and the normal $\mathbf{f}_\perp$ and tangential forces $\mathbf{f}_{//}$, in terms of the director angle ψ and the eigenvalues (μ_n,μ_m), with the purpose of elucidating the special features of biaxial nematic interfaces.

(A) Surface Free Energy Density F_s

In the biaxial state the surface free energy density F_s can be expressed as a sum of orientation dependent contributions and orientation-independent terms, as follows:

$$F_s = \omega_o + \omega_2\cos^2\psi + \omega_4\cos^4\psi \tag{21a}$$

$$\omega_o = (2\beta_{20})\mu_n^2 + (2\beta_{20} + \beta_{21} + \beta_{22})\mu_m^2 + (2\beta_{20})\mu_n\mu_m \tag{21b}$$

$$\omega_2 = (\beta_{11} + \beta_{21}(\mu_n + \mu_m) + 2\beta_{22}\mu_m)(\mu_m - \mu_n) \tag{21c}$$

$$\omega_4 = \beta_{22}(\mu_m - \mu_n)^2 \tag{21d}$$

It is seen that biaxiality through μ_m affects the magnitudes of the three coefficients ω_i; i=0,2,4. When the orientation dependent terms vanish ($\psi=\pi/2$), biaxial effects persist through ω_0. As a check of validity we note that when the state is uniaxial ($\mu_n = \mu_m$) in the (i_1,k) plane, the $(\mathbf{i}_1,\mathbf{k})$ plane, the orientation ψ plays no role.

(B) Surface Stress Tensor $\mathbf{t}^{se}$

The magnitudes of the normal (tensile) stresses are: $t_{11}^{se} = t_{22}^{se} = F_s(\mu_n,\mu_m,\psi)$, were F_s is given in eqn.(21). The extrema of the tensile stresses are those of the surface free energy density; their magnitudes and orientations ψ_e from the unit normal are:

$$\psi_e = 0, t_{11}^{se} = t_{22}^{se} = \omega_o + \omega_2 + \omega_4 \quad (22a, b)$$

$$\psi_e = \pi/2, t_{11}^{se} = t_{22}^{se} = \omega_o \quad (23a, b)$$

$$\psi_e = \cos^{-1}\sqrt{\frac{-\omega_2}{2\omega_4}}, t_{11}^{se} = t_{22}^{se} = \omega_o - \frac{\omega_2^2}{4\omega_4} \quad (24a, b)$$

Biaxial ordering affects the orientation of the oblique extremum and the magnitudes of all the extrema. Here and in the rest of the paper the special values of ψ, for stress extrema and zero stress, correspond to the first quadrant. Non-homogeneous tensile stresses may exist due to surface gradients of the director and/or surface gradients of the scalar order parameters.

In the planar director state ($\mathbf{l}=\mathbf{i}_2$), the nonzero bending stress is t_{13}^{se}, given by:

$$t_{13}^{se} = -\sin 2\psi\left[\omega_2 + 2\omega_4\cos^2\psi\right] \quad (25)$$

Since the bending stress is proportional to $\partial F_s/\partial(\cos\psi)$, it vanishes at the orientations ψ_o at which the tensile stresses achieve their extrema:

$$t_{13}^{se} = 0, \quad @ \quad \psi_o = 0, \quad \pi/2, \quad \cos^{-1}\sqrt{\frac{-\omega_2}{2\omega_4}} \quad (26a, b)$$

In between the orientations ψ_o corresponding to zero bending stress there are two orientations ψ_e for bending stress extrema:

$$\psi_e = \cos^{-1}\left\{\left[\frac{-(2\omega_2 - 6\omega_4) \pm \sqrt{(2\omega_2 - 6\omega_4) + 32\,\omega_2\,\omega_4}}{16\omega_4}\right]^{1/2}\right\} \quad (27)$$

Biaxiality again affects the magnitude and orientation of the bending stress extrema, thorugh μ_m. In addition zero bending stress and tension stress extrema coincide.

(C) Tangential Marangoni Force $\mathbf{f}_{//}$

Since F_s (μ_n, μ_m, cos ψ), the tangential Marangoni force $\mathbf{f}_{//}$ given by equation (17) gives:

$$\mathbf{f}_{//} = -\frac{\partial F_s}{\partial(\cos\psi)}\sin\psi\ \nabla_s\psi + \frac{\partial F_s}{\partial\mu_n}\nabla_s\mu_n + \frac{\partial F_s}{\partial\mu_m}\nabla_s\mu_m \quad (28)$$

where the partial derivatives $\partial F_s/\partial(\cos\psi)$, $\partial F_s/\partial\mu_n$, $\partial F_s/\partial\mu_m$ are obtained using equations (21). The nematic Marangoni force arises due to orientation and/or scalar order parameter surface gradients. Biaxiality affects the magnitudes of the three coefficients $\partial F_s/\partial(\cos\psi)$, $\partial F_s/\partial\mu_n$, $\partial F_s/\partial\mu_m$. The Marangoni force equation shows that non-homogeneous surface biaxiality at constant orientation creates tangential forces.

(D) Normal Force $\mathbf{f}_\perp$

Since F_s (μ_n, μ_m, cos ψ), the normal force $\mathbf{f}_\perp$ given by equation (16) gives:

$$\mathbf{f}_\perp = \left[2HF_s - \nabla_s \cdot \left(\frac{\partial F_s}{\partial \cos\psi} \mathbf{n} \right) - 2H \left(\frac{\partial F_s}{\partial \cos\psi} \cos\psi \right) \right] \mathbf{k} \qquad (29)$$

The surprising result is that even in the absence of curvature (H=0) nematic interfaces may display normal forces. The conditions that lead to zero normal force are H=0 and spatially homogeneous surface nematic ordering. For a planar interface, H=0, the normal force expression becomes:

$$\mathbf{f}_\perp = - \left[- \left[\frac{\partial}{\partial x_1} \left(\frac{\partial F_s}{\partial \cos\psi} \right) \right] \sin\psi - \left[\frac{\partial F_s}{\partial \cos\psi} \frac{\partial \psi}{\partial x_1} \right] \cos\psi \right] \mathbf{k} \qquad (30)$$

The presence of surface orientation gradients and/or scalar order parameter gradients generate normal forces even under zero curvature conditions. The influence of biaxiality is embodied by $\partial F_s/\partial(\cos\psi)$.

6. CONCLUSIONS

This paper derived the equations of capillary hydrostatics for interfaces between viscous isotropic fluids and nematic liquid crystals. As in all fluids the interfacial stress balance equation involves the surface divergence of the surface stress tensor. The presence of anisotropic elastic storage due to weak anchoring conditions gives rise to bending stresses, in addition to the normal stresses. Surface gradients of the normal stresses generate tangential forces that must be balanced by bulk Ericksen stresses if static conditions prevail. The tangential forces are similar to those generated by surface tension gradients caused by thermal gradients, concentration gradients, and electric charge gradients. Surface gradients of the bending stresses generate normal forces even in the presence of planar interfaces. For isotropic fluids at rest normal forces are generated by interfacial curvature, but for nematic-isotropic fluid interfaces the source is the anisotropic surface elasticity. In summary nematic-isotropic fluid interfaces are predicted to generate a range of effects originating from anisotropic surface elasticity, including bending stresses, tangential Marangoni forces, and curvature -independent normal forces. Mathematical expressions corresponding to biaxial nematics for anisotropic surface elasticity, surface stress tensor, Marangoni and normal forces are derived, characterized, and used to esatblish the main interfacial phenomena driven by nematic biaxial ordering.

Acknowledgements

Financial support of the Natural Sciences and engineering Research Council (NSERC) of Canada is gratefully acknowledged.

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